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PASSIVE LOCALIZATION OF PULSED SOUND SOURCES WITH A 2-HYDROPHONE ARRAY

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The passive localization of pulsed sound sources from differential arrival times of direct and surface-reflected arrivals at a pair of hydrophones is examined. Assuming a homogeneous ocean the errors in distance estimation are largest for source locations right below and at the side of the array and also close to the sea surface. The localization accuracy can be raised by increasing the hydrophone separation and the array depth and/or reducing the array angle with respect to the horizontal. In the case of stratified ocean environments a ray-theoretic localization approach is introduced taking acoustic refraction into account.

1. INTRODUCTION

The passive localization of vocalizing cetaceans is essential for behavioral studies as well as for the quantitative description of a number of characteristics of the produced sounds, such as source level and directionality. While there is a broad literature on sounds produced by various cetacean species, it is only recently that reliable range and source-level estimates of free-ranging cetaceans, sperm whales in particular, with passive means have been reported [1], [2], based on measurements of differential travel-times of click sounds at a large-aperture hydrophone array deployed from a number of independent platforms.

The use of a simple 2-element hydrophone array is examined here for localization and range estimation. The common use of such arrays is for bearing estimation using differential travel times of direct arrivals [3]. By exploiting both direct and surface-reflected arrivals an estimate for the animal location can be obtained. In the case of an homogeneous ocean simple closed-form expressions are derived for the location estimates. In the more general case where stratification is present a ray-theoretic localization approach is developed based on the assumption that the hydrophone spacing is much smaller than the source distance.

2. LOCALIZATION IN A HOMOGENEOUS MEDIUM

Let us consider an array of 2 hydrophones, 1 and 2, with separation L in a homogeneous medium with sound velocity c (Fig. 1). Let h be the depth of hydrophone 1 and a the array angle with respect to the horizontal. A Cartesian coordinate system (x,y,z) is adopted with origin at the location of hydrophone 1 and with the xz plane coinciding with the vertical plane containing the two hydrophones.

A pulsed signal originating at the source location (x_S,y_S,z_S). will reach the hydrophones 1 and 2 in times T_1 and T_2, respectively, following the direct propagation paths. The travel-times corresponding to the surface-reflected paths from the source to the hydrophones can be calculated as the direct-path travel times from the source to the mirror images 3 and 4 of the hydrophones 1 and 2 about the sea surface. In this connection the travel times of the surface-reflected arrivals at the hydrophones 1 and 2 are denoted by T_3 and T_4, respectively. The location vectors of the four hydrophones can be written in the form $\vec{r}_i = (x_i, 0, z_i)$, $i = 1,\ldots,4$

$$\vec{r}_1 = (0,0,0),$$
$$\vec{r}_2 = (L\cos a, 0, -L\sin a), \qquad (1)$$
$$\vec{r}_3 = (0,0,2h), \text{ and}$$
$$\vec{r}_4 = (L\cos a, 0, 2h + L\sin a).$$

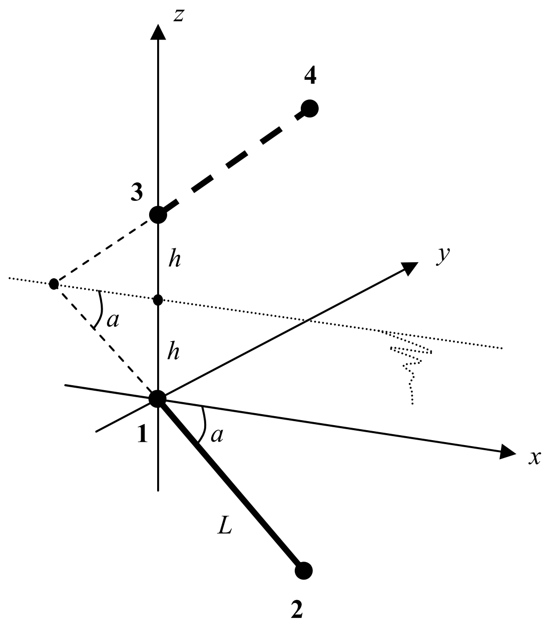

Fig. 1: A two-element hydrophone array (1,2) and its mirror image (3,4) about the sea surface. All hydrophones lie on the xz plane.

By referring all travel times to $T = T_1$, the arrival time at hydrophone 1, and defining the differential arrival times $t_i = T_i - T$, the following equations can be written

$$c^2 (T + t_i)^2 = (x_S - x_i)^2 + y_S^2 + (z_S - z_i)^2 \quad , \quad i = 1,\ldots,4 \qquad (2)$$

By subtracting the equation for $i = 1$ from the remaining equations the following system of linear equations is obtained

$$\begin{pmatrix} x_2 & z_2 & c^2 t_2 \\ x_3 & z_3 & c^2 t_3 \\ x_4 & z_4 & c^2 t_4 \end{pmatrix} \begin{pmatrix} x_S \\ z_S \\ T \end{pmatrix} = \begin{pmatrix} |\vec{r}_2|^2 - c^2 t_2^2 \\ |\vec{r}_3|^2 - c^2 t_3^2 \\ |\vec{r}_4|^2 - c^2 t_4^2 \end{pmatrix} \qquad (3)$$

By solving this system the unknowns x_S, z_S and T can be evaluated. The y-coordinate of the source location can be estimated from the relation $y_S = \pm\sqrt{c^2 T^2 - x_S^2 - z_S^2}$; with a 2-

hydrophone array configuration there are two symmetric solutions for y_S (left-right ambiguity), provided that $c^2T^2 > x_S^2 + z_S^2$.

The distance D of the source from hydrophone 1, is related with the reference travel time T by the relation $D = Tc$, and it is given by the following expression

$$D = -\frac{c}{2}\frac{ht_2^2 + (h + L\sin a)t_3^2 - ht_4^2}{ht_2 + (h + L\sin a)t_3 - ht_4} \tag{4}$$

Thus, the estimate for the distance D depends on the differential travel times (t_2, t_3, t_4), the array geometry (h, L, a) and the sound velocity c.

Errors in the measurement of the array depth and inclination as well as of the differential travel times will reflect in errors in the estimation of the source distance D. The first-order effect of this errors is given by

$$\delta D = \frac{\partial R}{\partial h}\delta h + \frac{\partial R}{\partial a}\delta a + \sum_{i=2}^{4}\frac{\partial R}{\partial t_i}\delta t_i \tag{5}$$

Assuming that the measurement errors are uncorrelated the variance of the error δD is expressed as

$$\left\langle \delta D^2 \right\rangle = \left(\frac{\partial R}{\partial h}\right)^2 \left\langle \delta h^2 \right\rangle + \left(\frac{\partial R}{\partial a}\right)^2 \left\langle \delta a^2 \right\rangle + \sum_{i=2}^{4}\left(\frac{\partial R}{\partial t_i}\right)^2 \left\langle \delta t_i^2 \right\rangle \tag{6}$$

Thus the rms error $\delta D_{RMS} = \sqrt{\left\langle \delta D^2 \right\rangle}$ for the estimated source distance can be calculated in terms of the underlying rms errors in array-geometry and travel-time measurement.

3. LOCALIZATION IN A HORIZONTALLY STRATIFIED MEDIUM

In a stratified ocean the acoustic propagation paths are curvilinear due to refraction and the previous analysis assuming straight-line propagation cannot be applied. In the framework of ray theory the geometry of the paths (rays) is governed by Snell's law [4]

$$\frac{\cos\varphi}{c} = const. \tag{7}$$

where $c = c(z)$ is the sound-speed profile and φ is the grazing angle of propagation. In the case of a linear sound-speed profile $dc/dz = b$ the ray becomes a circular arc with radius of curvature

$$R = \frac{c_0}{b\cos\varphi_0}, \tag{8}$$

where φ_0 is the launch angle of the ray and c_0 the sound speed corresponding to the launch depth z_0. According to Snell's law the propagation angle becomes zero at the depth $\hat{z}$ (turning depth) where the following condition holds

$$\frac{1}{c(\hat{z})} = \frac{\cos\varphi_0}{c_0} \tag{9}$$

At the turning points the ray changes direction of propagation in the vertical such that it heads towards areas of lower sound-speed values (less than $c(\hat{z})$). Using the above relations the ray geometry can be calculated.

The travel time along a ray, from depth z_0 to z_1 is expressed through the integral

$$\Delta T = \int_{z_0}^{z_1} \frac{dz}{\sin\varphi(z)c(z)} \qquad (10)$$

In the case of linear sound speed profile this integral can be evaluated explicitly and the travel time is given by the expression

$$\Delta T = \frac{1}{b}\left[\ln\frac{c_0 + \sqrt{c_0^2 - c^2\cos^2\varphi_0}}{c\cos\varphi_0}\right]_{c=c_0}^{c=c_1} \qquad (11)$$

This expression holds if there is no turning point within the interval of integration. Otherwise the integration has to be split into two parts, one from the beginning to the turning point and one from the turning point to the end.

Apart from travel-time calculation, a further point of interest for the localization problem is the calculation of the intersection point(s) between two circular rays. In general there can be two, one or no intersection points between two circles. The intersection location(s) can be easily found by solving a quadratic equation resulting from the equations of the two circles.

The expressions (8) and (11) hold for a linear sound-speed profile. In case of a piecewise linear profile these expressions can be applied for each layer of constant sound-speed gradient. Ray tracing can be carried out by applying continuity of propagation angles at the layer interfaces (provided that the sound speed is continuous) and reflection conditions at the sea surface.

Assuming that the separation of two hydrophones, A and B (Fig. 2a), is small compared with the source distance ($L \ll D$) the angle of arrival ψ at the two hydrophones is related with the differential arrival time $T_A - T_B$ through the relation

$$c\left(T_A - T_B\right) = L\cos\psi \ , \qquad (12)$$

where c is the sound speed at the location of the array. Using this relation the angle ψ of the direct arrivals at the hydrophones 1 and 2 can be calculated from the differential arrival time $T_2 - T_1$. Similarly, the angle ψ' of the surface-reflected arrivals can be calculated by taking the differential arrival time $T_4 - T_3$. The angles ψ and ψ' define two conical surfaces with axis of symmetry coinciding with the axis of the array.

If the case of a vertical array the problem becomes axisymmetric about the *z*-axis, in the sense that the grazing angles of the direct and surface-reflected arrivals are independent of the azimuthal direction. Otherwise, the two grazing angles depend on the azimuthal direction and they can be calculated from the intersection of the above two conical surfaces with the vertical plane corresponding to each azimuthal direction and containing the cone apex. The intersections of this plane with each conical surface are straight lines. In general there can be two, one or no intersections defining corresponding grazing angles. These can be obtained by solving a quadratic equation resulting from the equations of the conical surface.

Using the estimated angles of direct and surface-reflected arrivals for each azimuthal direction as initial conditions the geometry of the corresponding rays is calculated and intersections between direct and surface-reflected paths are found (Fig. 2b). For each intersection point (candidate source location) the travel time to the receiving array along the direct (T_D) and surface-reflected (T_{SR}) ray path is calculated and the time difference ($T_{SR} - T_D$) is compared to the differential arrival time ($T_3 - T_1$). The azimuthal directions and intersection points with $T_{SR} - T_D = T_3 - T_1$ are solutions of the localization problem.

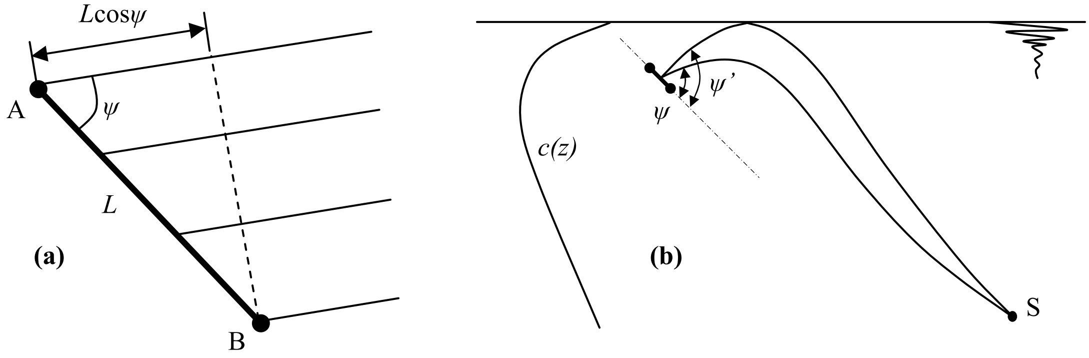

Fig. 2: a)Estimation of arrival angle assuming a distant source. b) Source localization in a stratified ocean environment.

4. NUMERICAL RESULTS

Some numerical results for passive source localization are given in this section. Starting with a homogeneous medium (c=1500 m/sec), Fig. 3 shows the rms error of the estimated distance D as a function of the source location for 4 array configurations assuming measurement errors (rms) of 0.01 msec for travel times and 0.1 m for depth. Assuming that the inclination of the array is estimated from the depths of the two hydrophones, the error in angle estimation is correlated with the measurement error for the depth of each hydrophone ($L\cos a \cdot \delta a_{rms} = \sqrt{2}\delta h_{rms}$). Fig. 3 presents the effect of the array depth (h), hydrophone separation (L), and array inclination angle (a). The errors of the estimated distance are quite significant in all cases and comparable with the true source distance [2]. The localization errors are largest right below and at the side of the array and also close to the sea surface. For fixed rms errors of depth and travel-time measurement the error in the estimated source distance can be reduced by increasing the array depth and the hydrophone separation and/or by decreasing the array inclination with respect to the horizontal. In this connection, a horizontal array offers the largest distance accuracy possible and a vertical array the smallest.

Fig. 4 presents an example of source localization in a stratified medium characterized by a bilinear sound-speed profile (Fig. 4a). The travel-time data were generated assuming a source at a range of 5 km from hydrophone 1 and at a depth of 1 km on the same vertical plane defined by the hydrophone array. The depth of hydrophone 1 was taken 50 m and the array inclination angle 30° from the horizontal. The separation between the two hydrophones was taken 30 m. The ray localization based on the actual sound speed profile reproduces accurately the azimuth direction, range and depth of the source (Fig. 4b). The source localization based on an average sound speed (homogeneous medium) results in an estimated source distance of 852.8 m, much smaller than the actual distance of 5099 m.

A localization scheme based on the homogeneous-medium assumption is anticipated to perform well for small ranges where the straight-line propagation is a good approximation. However, for larger ranges the refraction (ray bending) and the non-uniformity of the sound-speed distribution along the ray paths have significant effects on arrival times and must be accounted for.

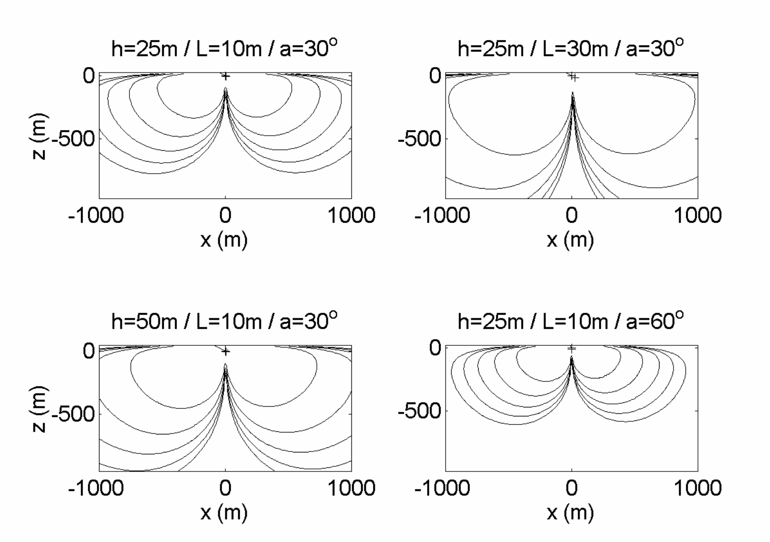

Fig. 3: The rms error in distance estimation in a homogeneous medium as a function of the source location for 4 array configurations. The 5 contours correspond to the levels 200m (inner), 400m, 600m, 800m and 1000m (outer).

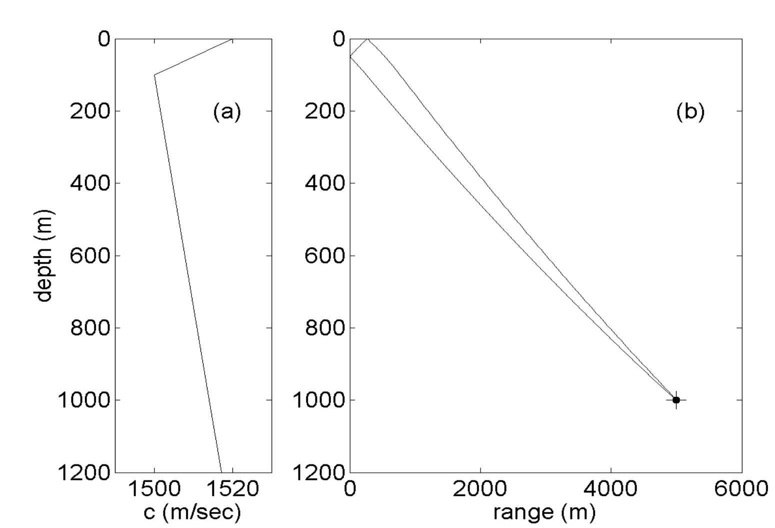

Fig. 4: Source localization in a stratified medium using ray tracing. a)Sound-speed profile. b) Intersection of direct and surface-reflected ray paths at the estimated source location (•); the true source location is marked by +.

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